Parametric Level-set Approach For Seismic Full Waveform Inversion

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Motivation
• Challenging to depict large bodies with hard exterior contrasts (salt diapirs, salt slabs, anhydrite or basement layers).
• Salt geometries are of particular interest because they often have hydrocarbon reservoirs on or underneath.
• Classic full waveform inversion (FWI) fails to recover salt geometry.

Full Waveform Inversion
Classical least squares formulation is defined as
\[
\min_m \left\{ \frac{1}{2} \| F(m) - d \|_2^2 \right\}
\]
where:
- \( m \) is the model vector defined in an infinite dimensional space. In general, \( m \) is discretized on a finite grid or mesh.
- \( d \) is the observed data from receivers when different shots are fired.
- \( F \) is the forward modelling operator which maps model to data space.

Prior Information
Generally, the information about the space of model vector is known. This kind of prior information is very helpful in optimization. Presently, we consider the distribution of background velocity, \( m_0 \) (sediment velocity) and salt velocity, \( m_s \) are known.

Model Representation
In general, the velocity model with salt can be represented as:
\[
m(x) = \begin{cases} 
m_s & \text{if } x \in \Omega \\
m_0 & \text{otherwise} \end{cases}
\]

Level Set Method
Represent the boundary of the domain \( \Omega \) as the zero level set of a function \( \phi(x) \). A smooth approximation of Heaviside function is applied on \( \phi(x) \) to distinguish between a background and salt structure.

Parametric Approach
• A level set function can be represented as the sum of finite radial basis functions with centers spread across the domain weighted by a factor.
• Parametric approach induces a dimensionality reduction in the problem.

The parametric level set is represented as follow:
\[
\phi(x) = \sum_{i=1}^{n} \alpha_i \| F(x) - x_i \|_2
\]
Hence, the parametric formulation leads us to
\[
\min_{\alpha} \left\{ \| F(m(x, \alpha)) - d \|_2 \right\}
\]

Radial Basis Functions
RBF are radially dependent functions which are typically used to approximate given functions.

They are categorized as infinite support and compactly supported RBF.

Figure 1 represents the different level set functions and the level set boundaries they form. Even disjoint level set can be formed with a single level set function.

Figure 2 shows the level set generated with a finite number of radial basis functions. It also shows the level set for each of the RBF.

Image Reconstruction of salt body
Salt reconstruction from FWI

Conclusions
• Parametric approach reduces the infinite dimensional problem into a finite dimensional one.
• Small number of RBF suffice to approximate level set of complex salt geometry (dimensionality reduction by almost a factor of 10).
• This approach is able to almost perfectly reproduce both anomaly and the basement reflector for FWI.

References

Acknowledgement
We acknowledge NWO-FOM, Shell Global Solutions and thank Profs. Felix Herrmann and Prof. Eldad Haber, Rajiv Kumar, Bas Peters from URC for their valuable advice and support.